Spatial-Domain Transformations: Something Old and Something New

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Summary

Gravity and magnetic transformations can be computed either in the wavenumber domain via a use of the Fourier Transform or in the spatial domain. In this work, I explain operational differences between the two types of transformations and explore their pros and cons. In particular, I apply to gravity and magnetic data some spatial-domain transformations that were originally developed in image processing and seismic interpretation.

Introduction

A gravity and magnetic transformation (or filtering) is a process of transforming observed gravity and magnetic anomalies into some new form. These transformations, in general, do not directly define the distribution of sources, but they often provide insights that help enhance a general understanding of the nature of sources, with the naked eye.

Transformations in the wavenumber domain (WD) have been very popular since the emergence of the computer. This fact lies largely in the efficiency of the Fourier Transform (FT). However, recent advancements in computational technologies make transformations in the spatial domain (SD) affordable. Most SD transformations of a practically sized grid (e.g., 1000 columns by 1000 rows) can be completed in a few minutes on a laptop computer. This prompts me to revisit SD transformations that were previously used in gravity and magnetics, which are something old; and to investigate some transformations that have not been applied, or have been applied inappropriately, to gravity and magnetics, which are something new.

In this work, I start with a description of operational differences between WD and SD transformations, then explain the advantages and limitations of each type of the transformations, and finally apply and examine some SD transformations. In applications, I focus on gridded data, not line data.

Operating on the entire grid or a small subset

The conventional WD transformation operates on the entire grid. An FT-based algorithm requires that a grid to be Fourier transformed (a) have a special number of columns and rows, e.g., a power of 2; (b) be full without holes and nulls; and (c) be periodic. A grid of a field survey result thus needs to be prepared. This preparation involves operations of expansion, interpolation, extrapolation and tapering; and will definitely introduce artificial data.

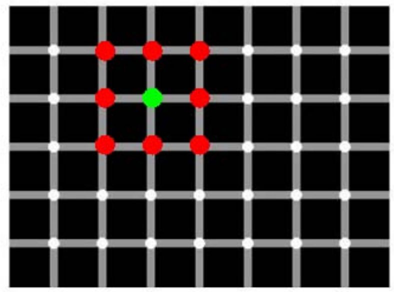


Figure 1. The nodes, one in green and eight in red, within a 3×3 window (subset) in a grid contribute to the single green node in a SD transform result.

A SD transformation operates on one subset at a time (Figure 1). It works in a moving window fashion. The user needs to define the window size, which is often an odd number. The window is often a square although it can be rectangular or circular.

Pros and cons of WD and SD transformations

In general, the SD transformation may have three advantages over the WD transformation: (1) has smaller edge effects, (2) is more robust to high-wavenumber (short-wavelength) noise in the data, and (3) allows for nonlinear transformations.

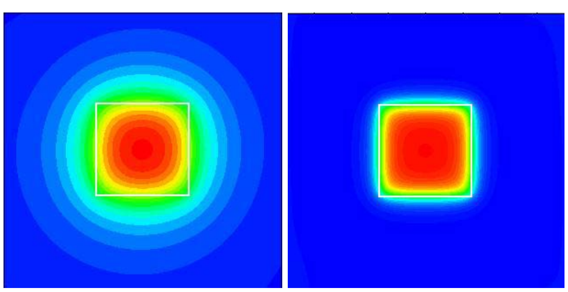
In a WD algorithm, all data points of the FT-prepared grid (including artificial data) will contribute to the transform result at any single point. The result is inaccurate near edges and may be inaccurate even at the very center of the survey area if the survey has an irregular shape and the transformation involves a phase shift. On the contrary, the SD computation uses a local subset at a time and thus has less pronounced edge effects. We don't need to expand the original grid if we can afford a loss of data near the edges.

Transformations such as derivatives have a transfer function proportional to wavenumber (magnitude). Noise in field data often largely sits in the high-wavenumber range, and is thus significantly amplified in a WD transform result. The locality of a SD operator makes such transformations relatively stable.

The Fourier transform is linear and WD transformations are thus linear. Clearly speaking, the WD transform result of the sum of grids *A* and *B* equals the sum of grids *A* and *B* transformed in the WD separately. The linear filtering process is called convolution. For this reason, linear SD filtering is referred to as convolving a grid with a mask. This mask is thus called a convolution mask (or kernel). The coefficients of the mask are predefined for a particular linear filter. A linear filter replaces the value at the center of the moving window with a weighted average of all grid values within the window (Figure 1), with the mask coefficients as the weighting factors. For example, the coefficients of the mean filter are all ones (divided by the number of coefficients).

Nonlinear SD filters also slide a mask past a grid. However, the filtering operation is based conditionally on the values of the grid nodes in the neighborhood under consideration, and they do not explicitly use coefficients in the sum-of-products manner. For example, filters involving the use of the median or variance are nonlinear and cannot have predefined coefficients for their masks.

Gravity and magnetic anomalies reflect primarily lateral variations of rock densities and susceptibilities. Like in the science of image processing, edges (or boundaries) play an important role in our perception and qualitative interpretation of gravity and magnetic anomalies. As such, it is important to be able to suppress noise and smooth images without disturbing the sharpness and, if possible, the position of edges. Nonlinear filters are much better choices for this task than linear ones. A group of so-called edge-preserving (or edge-enhancing) smoothing filters have been specifically developed for this application.



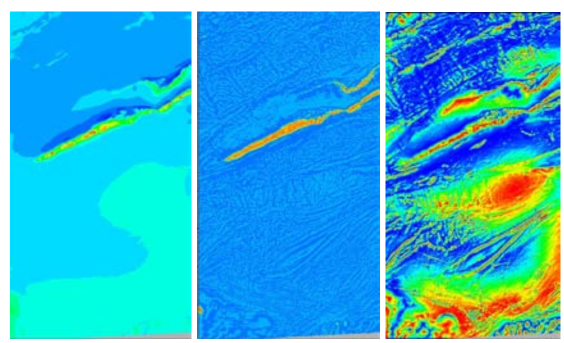
(a) (b)

Figure 2. Theoretical (a) gravity and (b) vertical gravity derivative (or magnetic for a vertical magnetization) due to a cube with its horizontal projection shown as the white square. The grid has 301 columns by 301 rows. Historical FVD operators have a size of 15×15 or smaller and are unable to produce a FVD of (a) that matches (b) accurately. In all color images of this work, low values are in blue and high in red, and the color sweeps are linearly stretched to the full data range in each image.

Even so, SD filters cannot replace WD filters completely. For example, it is still far preferable to compute the magnetic reduction-to-the-pole transformation in the WD. The phase operator that depends on the direction of the magnetization, the phase operator that depends on the direction of the magnetic observation, and the amplitude operator are decoupled with one another in the WD. This is not the case in the SD.

It is also inaccurate to compute a SD transformation that involves surface integration, which requires the full extent of anomalies. In practice, we use a mask with a limited size that may only partially catch the long-wavelength information. These transformations also include continuation and the first vertical derivative (FVD).

The FVD indeed involves an integration process because observations are routinely made on one surface. The FVD represents the difference between results on two vertically separated surfaces. It requires a continuation computation to obtain results on the second surface. I have tested three historical SD operators. Among them the operator of McGrath (1974) has the largest size, 15×15. I have found that none produces an acceptable result for my test model (Figure 2). Instead, a hybrid approach works well. This is the so-called "the FVD as a function of the SVD (second vertical derivative)", which was proposed in Ackerman and Dix (1955) and has been used by Fedi and Florio (2001). In my implementation, I compute the vertical integration (VI) of an input grid in the WD, the SVD of the VI result in the SD by a Laplacian operator: the final result is the FVD.



(a) (b) (c)

Figure 3. (a) The total magnetic intensity anomaly (values ranging from –700 nT to 1000 nT) of the Gaspé belt in the northern Québec Appalachians, (b) its AGC transform result for a window size of 3×3, and (c) the tilt angle.

Nonlinear transformations for enhancement of subtle features

In gravity and magnetics, several nonlinear transformations have been developed to enhance subtle low-amplitude features. They include automatic gain control (AGC) (Rajagopalan and Milligan, 1995), logarithmic transformation (Gonzalez and Woods, 2001, p. 79; Morris, et al., 2001), and tilt angle (Miller and Singh, 1994). The first two are pure SD transformations, and the last one is not when its required FVD is not measured directly. Again, the FVD purely calculated in the SD is often inaccurate.

The AGC and the tilt angle don't preserve absolute amplitude information. However, the logarithmic transformation does. All three transformations are effective particularly when the input anomalies contain a wide variety of wavelengths and a wide range of magnitudes (Figure 3).

Edge-preserving smoothing filters (EPSF)

The median filter may be the most famous nonlinear filter as well as the simplest EPSF. First, the median is a more robust average than the mean — a single very unrepresentative node in a neighborhood will not affect the median value significantly. It is a very effective tool for removing spiky (or impulse) noise. Second, the median value must actually be the value of one of the nodes in the neighborhood; and the median filter does not create new unrealistic nodal values when the filter straddles an edge. For this reason the median filter is much better at preserving sharp edges while smoothing signals than the mean filter.

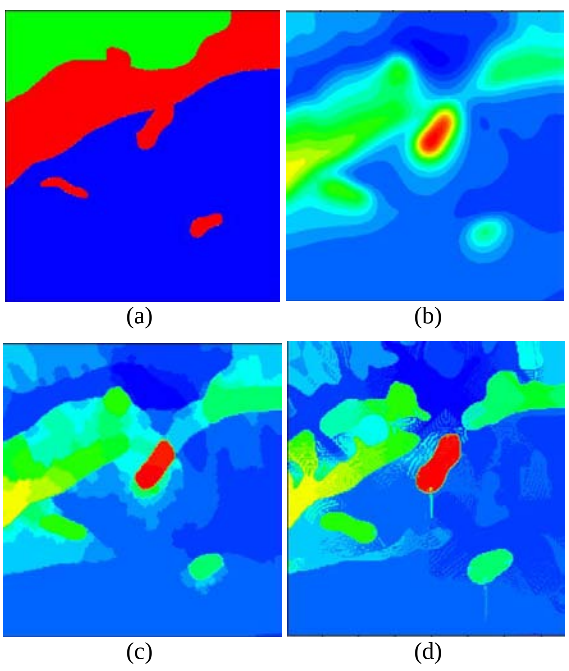


Figure 4. A Bishop model test. (a) Magnetic susceptibility variations of the model, (b) magnetic responses for a vertical magnetization, (c) the SNN transform result for a window size of 5×5 and 20 iterations, and (d) the terracing result after 20 iterations.

In image processing, people have developed other advanced nonlinear EPSFs that include the Kuwahara filter (Kuwahara et al, 1976), the mean of least variance (MLV) filter (Schulze and Pearce, 1994), and the symmetric nearest neighbor (SNN) filter (Hardwood et al, 1987). The first two compare different neighboring windows and take the average value of the window that has the least variance. The last one compares directly values of symmetric pairs and is thus faster. Luo et al (2002) have applied the MLV filter to seismic data, and I have implemented all three in gravity and magnetic interpretation.

EPSFs are similar to the terracing filter for gravity and magnetic interpretation (Cordell and McCafferty, 1989). In terracing, local curvature is defined as second-order derivatives and computed by the Laplacian operator. The field at the center of a window is increased, decreased, or unchanged depending on the algebraic sign of local curvature. In practice, an EPSF or the terracing filter needs to be applied repeatedly (e.g., 20 times or iterations) in order to achieve an acceptable boundary-sharpening effect. As demonstrated in Figures 4, the terracing filter is often more sensitive to noise or suffers a stronger instability of numerical computations.

Coherence filter

Coherence filters measure similarity. Bahorich and Farmer (1995) published the first result of applying the coherence method to seismic data. Faults in seismic reflection data show least similarity, i.e., least coherence. In principle, coherence is applicable to mapping of faults and discontinuities in a gravity or magnetic anomaly grid. However, it should be appreciated that gravity and magnetic data don't generally contain the sample density and detailed character that seismic data does, and hence don't have the same mapping ability as seismic data.

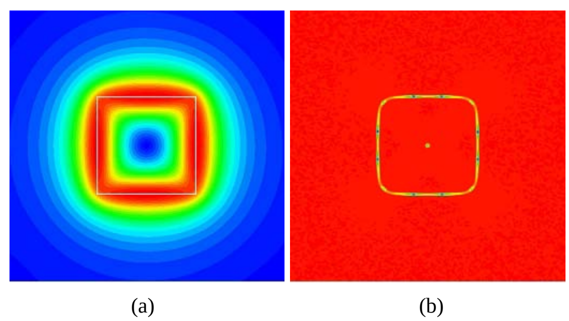


Figure 5. (a) The horizontal gradient of the gravity in Figure 2a; and (b) the cross-correlation based coherence, for a window size of 3×3, of (a). Low coherence values outline the boundary of the cube (i.e., the white square in (a)) very well.

Spatial-domain transformations

I have programmed two algorithms for the coherence computation. The Cross-Correlation-based Coherence (CCC) is the original in Bahorich and Farmer (1995) and faster. Gunn et al (1997) computed the CCC of aeromagnetic data in two 7×7 windows, but this is only the old cross-correlation for an automatic trend analysis of a gravity or magnetic grid (Agarwal and Kanasewich, 1971). My understanding is that a CCC should be rightfully computed by cross-correlating the central window with the neighboring eight ones, i.e., in all directions. The eigenvalue-based coherence computation is more time-consuming but often more robust to noise (Gersztenkorn and Marfurt, 1999). In practice, I find that for boundary analysis, the coherence of the horizontal gradient of the gravity (or pseudogravity of magnetic) anomaly particularly further enhances the detectability of the widely used horizontal gradient method itself (Figure 5).

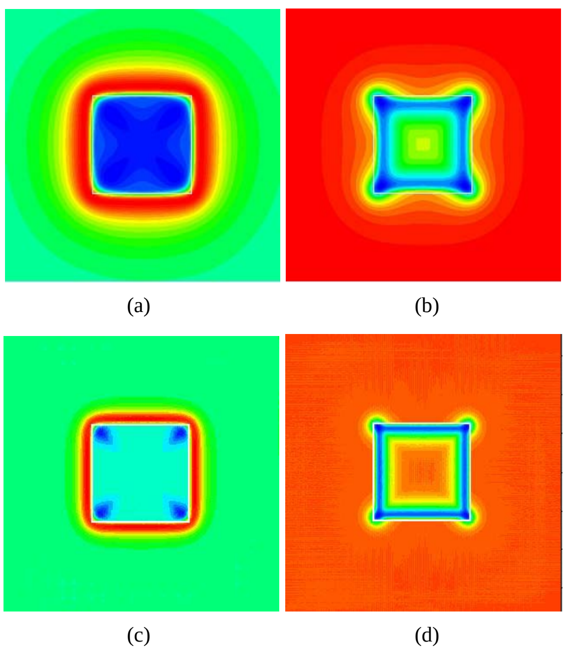


Figure 6. Curvature attributes of gravity and magnetic responses due to a cube. (a) The most positive curvature and (b) the most negative curvature of gravity responses in Figure 2a, (c) the most positive and (d) the most negative of magnetic in Figure 2b.

Curvature attributes

Curvature may be defined as a second-order derivative of a curve. Roberts (2001) has introduced curvature to seismic systematically and defined many different curvature attributes: the mean, Gaussian, maximum, minimum, most positive, most negative, dip, strike, contour curvatures, the curvedness and the shape index. The gravity anomaly uses units of mGal and magnetic anomaly nT, not milliseconds or meters. For qualitative analysis of gravity and magnetic data, we always expect that the shape (if not the magnitude) of a transform result is independent of the horizontal grid spacing and units. As a result, only the most positive, most negative, and contour curvature attributes can be applied to gravity and magnetic data.

Roberts (2001, p. 99) wrote: “Curvature contains the added dimension of shape, allowing faults, fault orientations and fault geometries to be delimited, as well as discriminating faults from other linear surface features”. In gravity and magnetics, Hansen and deRidder (2006) applied the most positive curvature for linear feature analysis and the most negative curvature for estimation of source depth.

For lineament analysis, I have tested the most positive and most negative curvatures on many gravity and magnetic data sets. None of the results is as interpretable as those of the horizontal gradient or the coherence. Figure 6 shows such a model test example.

Conclusions

The WD transformation imposes a prerequisite for the input grid and works on the entire grid, while the SD transformation works by applying a moving window with a limited size. The WD transformation thus suffers from more pronounced edge effects. As well, the WD is often less robust to noise and doesn’t allow nonlinear transformations. However, it is very difficult to compute accurately in the SD a transformation that involves a phase shift or surface integration operation.

Transformations that are better performed in the SD and useful for gravity and magnetic interpretation may include (a) the AGC filter and the tilt angle that highlight low-amplitude features and thus resolve deep and shallow sources equally well; (b) EPSFs for edge enhancement and lineament analysis; and (c) the coherence filter that can further discriminate discontinuities. All are nonlinear transformations. I have implemented and tested all and found that they are powerful tools in qualitative interpretation.

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