

## **The quest for the perfect gravity anomaly: Part 1 — New calculation standards**

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### **Summary**

The North American gravity database together with databases from Canada, Mexico, and the United States are being revised to improve their coverage, versatility, and accuracy. An important part of this effort is revision of procedures and standards for calculating gravity anomalies taking into account our enhanced computational power, modern satellite-based positioning technology, improved terrain databases, and increased interest in more accurately defining different anomaly components. The most striking revision is the use of one single internationally accepted reference ellipsoid for the horizontal and vertical datums of gravity stations as well as for the computation of the theoretical gravity. The new standards hardly impact the interpretation of local anomalies, but do improve regional anomalies. Most importantly, such new standards can be consistently applied to gravity database compilations of nations, continents, and even the entire world.

### **Introduction**

Historically, gravity reductions in geophysics were applied to the observed gravity after the meter readings were corrected for effects such as meter drift, tide, Eötvös, loop ties, etc. Gravity reductions comprised the latitude, free-air, complete Bouguer, and isostatic reductions. The complete Bouguer reduction itself included the simple infinite horizontal slab, curvature, and terrain reductions. Sea level (or the geoid) was the vertical datum for gravity stations and Digital Terrain Models (DTMs). The elevation (or orthometric height) relative to sea level was used throughout the reductions.

Methods of anomaly analysis and interpretation have changed dramatically with new theoretical developments especially in filtering, enhancement, and inversion techniques that take advantage of high-speed computers. However, the reduction procedures, with minimal modification, continue to be used in local surveys, in regions of rugged topography, and in regional anomaly compilations. As more accurate gravity anomalies have become of interest, especially regional anomaly compilations and observations in rugged topography, modifications to reduction procedures were investigated (e.g., Olivier and Hinze, 1986; LaFehr, 1991a, 1991b; Chapin, 1996a; Li and Götze, 2001) and implemented on a limited basis, but they have not been used generally and have not been employed in preparing national and North

American gravity databases. Use of these modifications has received impetus from the availability of improved terrain and geoid databases, enhanced computational power, and increasing use of global positioning system (GPS) technology to establish the location and heights of gravity stations. As a result when plans were being made to revise the North American gravity database, the decision was made to upgrade its quality not only in the number and coverage of observations by including surveys that were made available since the previous compilation, but also using improved procedures to increase the accuracy of anomaly values.

The new standards for the revised databases (Hinze et al., 2005) are based on internationally accepted procedures, protocols, equations, and parameters, but in several respects differ significantly from the standard procedures described in current textbooks and used in available databases. The revisions improve the precision and accuracy of anomalies.

### **Datums**

The new standards recommend a common ellipsoid, the Geodetic Reference System 1980 (GRS80) recommended by the International Union of Geodesy and Geophysics, for three necessary datums in gravity methods: the horizontal, vertical, and theoretical gravity. In practice, the GRS80, WGS84 (World Geodetic System 1984) used for the GPS, and ITRF (the International Terrestrial Reference Frame) produce no significant differences in the gravity anomaly calculation and results. Using one single ellipsoid has many advantages.

#### Horizontal datum

It doesn't use the nationally selected horizontal control net. This avoids problems in positioning of stations across national boundaries caused by varying horizontal datums used by the nations as well as in calculating the latitude effect of gravity due to a reference ellipsoid.

#### Vertical datum and the geophysical indirect effect

Using the GRS80 ellipsoid as the vertical datum has three benefits. 1) It avoids problems caused by inconsistencies among national vertical datums. 2) It directly uses GPS heights relative to the ellipsoid without the need to correct for geoidal deviations that may be inadequately determined. 3) It eliminates the need for the geophysical indirect effect calculation and removal (e.g., Li and Götze, 2001).

## New standards for calculating gravity anomalies

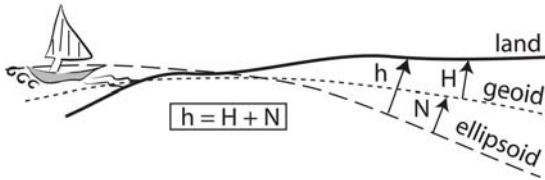


Figure 1. Cartoon showing (a) the three surfaces: topographic, ellipsoidal, and geoidal and (b) the three heights: the ellipsoidal height  $h$ , the elevation  $H$ , and the geoidal height  $N$ .

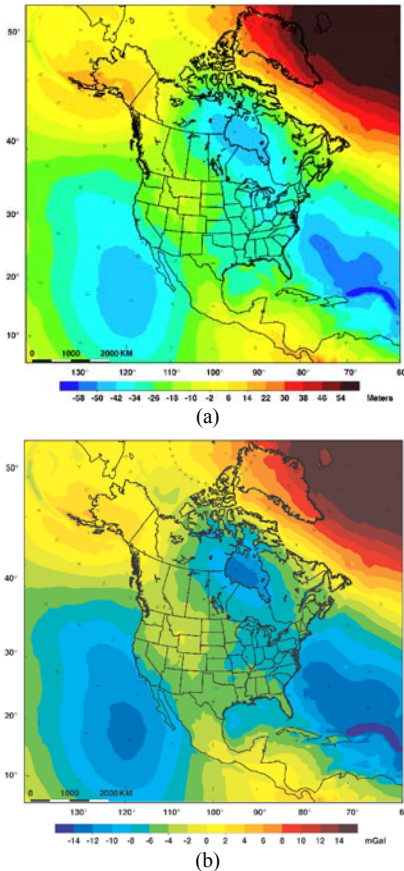


Figure 2. For the North America, (a) geoidal undulations from the EGM96, and (b) the geophysical indirect effect of the traditional free-air reduction using the elevation not the ellipsoidal height.

Figure 1 displays the three surfaces: topography, ellipsoid, and geoid; and the relationship among the ellipsoidal height, elevation, and geoidal undulation. Historically, the geoid or sea level, not the ellipsoid, is used as the vertical datum; and the elevation, not the ellipsoidal height, is used for establishing the height of the station and topographic relief (DTMs) and thus for computation of gravity reductions and anomalies. This evidently introduces a discrepancy since the theoretical gravity is calculated of an

ellipsoid, never of a mass body enclosed by any geoid. The gravity difference caused by this discrepancy or use of two different heights is just the geophysical indirect effect.

The indirect effect is negligible in local surveys because geoidal undulations vary slowly. However, the effect can interfere with interpretation of long wavelength ( $> 100$  km) anomalies that are of increasing interest in continental-scale mapping and neotectonic activity studies. The indirect effect attains magnitudes of up to  $\pm 30$  mGal worldwide. Figure 2 shows geoidal undulations and the indirect effect of the traditional free-air reduction in the North America.

### Gravity datum

Gravity values are referenced to the International Gravity Standardization Net 1971 (IGSN71) (Morelli et al., 1974). However, published IGSN71 values include a correction that removes the average part of the tidal force. This correction is inappropriate (Heikkinen, 1979) and can be removed from IGSN71 values by adding the latitudinal dependent correction in mGal as given by (Uotila, 1980)

$$\Delta g_H = 0.0371(1 - 3 \sin 2\varphi) \quad (1)$$

where  $\varphi$  is the latitude (South or North) of the station.

### **From observed gravity data to anomalies**

Raw gravity data are affected by a wide variety of sources of varying amplitudes, periods and wavelengths that generally mask gravity variations of geologic or geophysical interest. “Reductions” and “corrections” are designed to minimize these extraneous effects. Correction does not imply that errors are present in the data and reduction does not suggest that the data are reduced to a common vertical datum. In today’s understanding, neither correction nor reduction is an appropriate term. A geophysical gravity anomaly is the difference between the observed gravity and the modeled or predicted value of gravity at the station. This modeling or prediction is to estimate gravity effects of known, unwanted sources. It is part of the process of interpreting the observed gravity.

### Theoretical gravity of the reference ellipsoid

The most dominant unwanted effect is the theoretical gravity at the station height of a reference ellipsoid that accounts for the mass, shape, and rotation of the Earth. This effect can be calculated by a single closed-form formula (e.g., equation (A-2) of Li and Götze, 2001), but is routinely divided into two estimates: the latitude effect on the ellipsoidal surface and the height effect at the observation altitude.

The Somigliana closed-form formula (Somigliana, 1930), not the International Gravity Formula, is used to calculate the theoretical gravity  $g_r$  on the ellipsoidal surface

## New standards for calculating gravity anomalies

$$g_T = g_e (1 + k \sin^2 \varphi) / \sqrt{1 - e^2 \sin^2 \varphi} \quad (2)$$

where the GRS80 reference ellipsoid has the following values:  $g_e = 978032.67715$  mGal;  $k = 0.001931851353$ ;  $e^2 = 0.0066943800229$ .

The new standards use the second-order approximation formula (Heiskanen and Moritz, 1969) to estimate the difference (in mGal) between the theoretical gravity at ellipsoidal height  $h$  (meters), not at elevation  $H$ , and the one on the ellipsoidal surface. For the GRS80, the formula is

$$g_h = -(0.3087691 - 0.0004398 \sin^2 \varphi)h + 7.2125 \times 10^{-8} h^2$$

Depending on the latitude, the difference between the first- and second-order estimates can be as large as several milligals at heights of several thousand meters (Li and Götze, 2001).

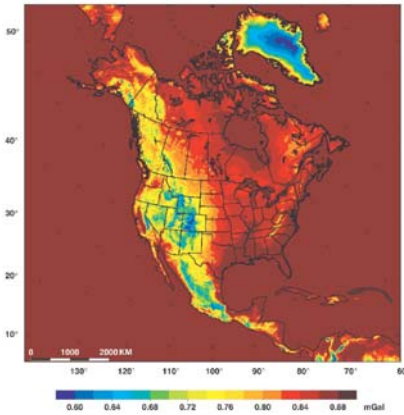


Figure 3. The atmospheric effect in the North America.

### Atmospheric effect

The gravity effect of an atmospheric mass model above a station can be calculated to the 0.01 mGal accuracy up to a height of 10000 m with the equation (Wenzel, 1985)

$$g_{atm} = 0.874 - 9.9 \times 10^{-5} h + 3.56 \times 10^{-9} h^2 \quad (4)$$

where the gravity effect is given in mGal and the height in meters. This effect should be subtracted from the theoretical gravity or added to the observed gravity because the reference ellipsoid already includes this mass but its gravity effect is not measured assuming that the atmosphere consists of homogeneous spherical shells. Figure 3 shows the atmospheric effect in North America.

### Bouguer effect

Rigorously speaking, the Bouguer effect is the gravity response, at an observation location, of topographic (including bathymetric) relief extending to the entire Earth relative to the reference ellipsoid. Considering the relative

accuracy and computational efficiency, however, one estimates the Bouguer effect of varying topography to a limited extent and relative to a sphere or even a flat plane. Use of a spherical not an ellipsoidal reference introduces significant errors at wavelengths of thousands of kilometers or longer. On the other hand, the difference between a spherical reference and a planar reference becomes non-negligible at wavelengths of tens of kilometers.

In a spherical coordinate system, it is possible to calculate the complete Bouguer effect in one step. This is done by dividing the entire region radially into circular zones and laterally into segments. A closed-form formula can be used to evaluate the gravity effect of such a topographic element.

In the new standards, we recommend a spherical model with a correction radius of 166.7 km and break the complete Bouguer effect into two parts: (1) the effect of a spherical cap and (2) terrain and bathymetric effect. Compared to the horizontal slab, the spherical cap considers the Earth's curvature, and its gravity effect can be estimated by a closed-form formula (LaFehr, 1991b)

$$g_{SC} = 2\pi G \sigma [(1 + \mu)h - \lambda R] \quad (5)$$

where  $\mu$  and  $\lambda$  are dimensionless coefficients;  $R = R_o + h$  where  $R_o$  is the mean radius of the Earth,  $h$  the height relative to the ellipsoid; and  $\sigma$  is the density of the material making up the spherical cap. Figure 4 shows the curvature effect, i.e., the difference between the spherical cap effect and the horizontal slab effect, in the North America.

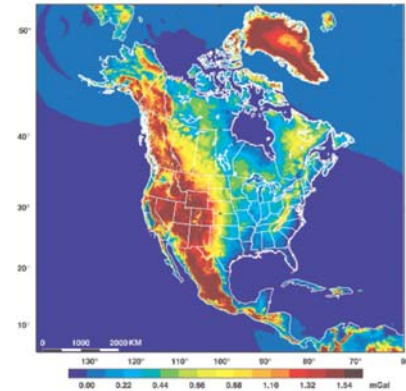


Figure 4. The curvature effect in the North America.

### Terrain and bathymetric effect

The subtraction of terrain effects from observed gravity minimizes rugged topographic effects that may cause errors in the gravity anomalies of tens of milligals. Efficient and effective computation of these effects requires comprehensive DTMs and computational power that are only now becoming generally available.

## New standards for calculating gravity anomalies

The US and North American gravity database processing uses a 3-part procedure for computing terrain (including lakes and glaciers) effects. The first part, the near-station effect involves topographic information collected in the field to a distance of about 100 m from the station. The second portion uses local high-resolution DTMs (e.g., 10 and 30 m grid spacing) to calculate terrain effects to a distance of 895 m, which includes Hammer's Zone F. The final part involves computation of the terrain effects (e.g., Godson and Plouff, 1988) from 895 m to 166.7 km based on 15 second and 1- and 3-minute DTMs and accounting for the Earth's curvature beyond 14 km.

We anticipate that long-term modifications to these procedures will count the terrain and bathymetric effects to 500 km and beyond using 2- and 5-minute terrain and bathymetric grids. Eventually, a global treatment will be available for terrain effects including densities that can be modified for surface geological variations.

### *Isostatic compensation effect*

The gravity effect of the isostatic compensation is derived from a geological model based on the theory of isostasy that regional topographic variations are compensated by density changes of the lithosphere such that at some depth the Earth is in hydrostatic equilibrium. We calculate the isostatic compensation effect in a manner similar to the terrain effect using the Airy-Heiskanen model (Heiskanen and Vening Meinesz, 1958) with a modified version of the procedure of Jachens and Roberts (1981) that assumes local isostatic compensation.

The correction for varying depth to a hypothetical crust-mantle boundary caused by differential topographic or bathymetric loads above or below the ellipsoid assumes a density variation of 0.3 g/cm<sup>3</sup> and a crustal thickness of 30 km for sea surface elevation based on average global crust/upper mantle density contrast and crustal thickness (Christensen and Mooney, 1995). Topography is modeled assuming a continental crustal density of 2.67 g/cm<sup>3</sup> using 3-minute elements of topography to 166.7 km plus interpolated values from 166.7 km to 180° from tables in Karki et al. (1961). In the future it is desirable to perform isostatic effect calculations using a method in the space domain consistent with the procedures for calculating the terrain effects using global DTMs.

### **Gravity anomalies**

Anomalies are differences between the observed gravity corrected for temporal variations and the modeled gravity. Different types of anomalies reflect variations in the components used in defining the modeled gravity at the station. The models change depending upon the use of the anomaly in geophysical studies.

The free-air anomaly represents the difference between the observed gravity and the modeled one taking into account the theoretical gravity due to the GRS80 ellipsoid and the atmospheric effect, all at the station location. The Bouguer anomaly additionally removes the effects of a spherical cap and terrain models. The isostatic anomaly further subtracts the effect of an isostatic compensation model.

Free-air gravity anomalies are frequently employed in modeling and map interpretation in marine areas without significant bathymetric variations. Bouguer anomalies are used primarily in the modeling and map interpretation of land gravity anomalies. Isostatic anomalies are used in interpreting regional gravity anomalies especially in map form, although care must be taken to consider the potential problems in the calculation of this anomaly derived from the assumption of isostatic compensation associated with local topographic variations (Simpson et al., 1986).

### **Conclusions**

New standards use a single reference ellipsoid as the vertical and horizontal datums as well as for the theoretical gravity calculation. Thus, it is advisable to informally recognize that anomalies calculated using the revised procedures with the adjective ellipsoidal preceding the gravity anomaly. New standards and procedures, being implemented in compilation of the revised North American and associated national gravity databases, will improve the accuracy and geophysical utility of gravity anomalies.

Changes in the anomaly calculation procedures minimize errors due to terrain, Earth curvature, vertical gradient, and atmospheric mass effects. As a result, on a relative basis the anomaly values of nearby stations are changed minimally in most regions except in areas of rugged topography where relative changes may exceed 1 mGal, but changes of up to tens of milligals may be present in absolute values.

We anticipate that not only will the North American gravity databases use the new standards, but also they will increasingly become the norm in the computation of gravity anomalies from observed gravity.

### **Acknowledgement**

We thank other members of the North American Gravity Database Committee for making comments and suggestions to the new standards.

### **References**

Hinze, W. J., et al., 2005, New standards for reducing gravity observations: The North American gravity database, *Geophysics*, **70(4)**, J25-J32. (Refer to Hinze et al. (2005) for details of the other references.)